

A Scenario for Spontaneous CP Violation in $SUSY\ SO(10)^*$

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Abstract

A scenario is suggested for spontaneous CP violation in non- $SUSY$ and $SUSY\ SO(10)$. The idea is to have a scalar potential which generates spontaneously a phase, at the high scale, in the VEV that gives a mass to the RH neutrinos. The case of the minimal renormalizable $SUSY\ SO(10)$ is discussed in detail. It is demonstrated that this induces also a phase in the CKM matrix. It is also pointed out that, in these models, the scales of Baryogenesis, Seesaw, Spontaneous CP violation and Spontaneous $U(1)_{PQ}$ breaking are all of the same order of magnitude.

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Introduction

There are three manifestations of CP violation in Nature:

- 1) *Fermi scale CP violation* as is observed in the K and B decays [1]. This violation is induced predominantly by a complex mixing matrix of the quarks (CKM).
- 2) *The cosmological matter antimatter asymmetry (BAU)* is an indication for high scale CP violation[2]. In particular, its most popular explanation via leptogenesis[3] requires CP breaking decays of the heavy right-handed (RH) neutrinos.
- 3) *The strong CP problem* called also the QCD Θ problem [4] lies in the non-observation of CP breaking in the strong interactions while there is an observed CP violation in the interaction of quarks.

Where is CP violation coming from? Is there one origin to all CP breaking phenomena?

It was already suggested [5][6] that a spontaneous violation of CP [7], at a high scale, via the spontaneous generated phase of the VEV that gives mass to the RH neutrinos, can be the origin of CP violation.

Why spontaneous violation of CP ?

- 1) It is more elegant and involves less parameters than the intrinsic violation in terms of complex Yukawa couplings. The intrinsic breaking becomes quite arbitrary in the framework of $SUSY$ and GUT theories.
- 2) Solves the $SUSY$ CP violation problem (too many potentially complex parameters) as all parameters are real.
- 3) Solves the strong CP problem at the tree level for the same reason.

For good recent discussion of spontaneous CP violation ($SCPV$), with many references, see Branco and Mohapatra [8].

Why CP breaking at a high scale ?

1) Needed to explain the BAU . Especially in terms of leptogenesis, i.e. CP violating decays of heavy neutrinos, it is mandatory.

2) $SCPV$ cannot take place in the standard model (SM) because of gauge invariance. Additional Higgs must be considered and those lead generally to flavor changing neutral currents ($FCNC$). The best way to avoid these is to make the additional scalars heavy [8].

3) The scale of CP violation can then be related to the seesaw scale as well as to the $U(1)_{PQ}$ [9] breaking scale, i.e. the “axion window” [4].

The conventional $SO(10)$

Let me start by revising the *renormalizable non-SUSY* $SO(10)$ and a possible $SCPV$ [6]. Conventional $SO(10)$ requires intermediate gauge symmetry breaking (I_i)[10] to have gauge coupling unification.

$$SO(10) \longrightarrow I_i \longrightarrow SM = SU_C(3) \times SU_L(2) \times U_Y(1) .$$

Most models involve an intermediate scale at $\approx 10^{12} GeV$ for:

- Breaking of $B - L$
- The masses of RH neutrinos
- CP violation responsible for leptogenesis (BAU)

$SO(10)$ fermions are in three **16** representations: $\Psi_i(\mathbf{16})$.

$$\mathbf{16} \times \mathbf{16} = (\mathbf{10} + \mathbf{126})_S + \mathbf{120}_{AS} .$$

Hence, only $H(\mathbf{10})$, $\overline{\Sigma}(\overline{\mathbf{126}})$ and $D(\mathbf{120})$ can contribute directly to Yukawa couplings and fermion masses. Additional Higgs representations are needed for the gauge symmetry breaking.

One and only one VEV $\overline{\Delta} = \langle \overline{\Sigma}(1, 1, 0) \rangle$ can give a (large) mass to the RH neutrinos via

$$Y_\ell^{ij} \nu_R^i \overline{\Delta} \nu_R^j$$

and so induces the seesaw mechanism. It breaks also $B - L$ and $SO(10) \rightarrow SU(5)$.

Spontaneous CP violation in conventional $SO(10)$

$\overline{\Sigma}(\overline{126})$ is the only relevant complex Higgs representation. Its other special property is that $(\overline{\Sigma})_S^4$ is invariant in $SO(10)$ [11]. This allows for a $SCPV$ at the high scale, using the scalar potential: [6]

$$V = V_0 + \lambda_1 (H)_S^2 [(\overline{\Sigma})_S^2 + (\overline{\Sigma}^*)_S^2] + \lambda_2 [(\overline{\Sigma})_S^4 + (\overline{\Sigma}^*)_S^4] .$$

Inserting the $VEVs$

$$\langle H(1, 2, -1/2) \rangle = \frac{v}{\sqrt{2}} \quad \overline{\Delta} = \frac{\sigma}{\sqrt{2}} e^{i\alpha}$$

in the neutral components, the scalar potential reads

$$V(v, \sigma, \alpha) = A \cos(2\alpha) + B \cos(4\alpha) .$$

For B positive and $|A| > 4B$ the absolute minimum of the potential requires

$$\alpha = \frac{1}{2} \arccos \left(\frac{A}{4B} \right) .$$

This ensures the spontaneous breaking of CP [12].

However, Φ^4 cannot be generated from the superpotential in renormalizable $SUSY$ theories and a different approach is needed there.

Renormalizable $SUSY SO(10)$ models

Became very popular recently [13] [14] [15] [16] due to their simplicity, predictability and automatic R -parity invariance (i.e. a dark matter candidate).

I will limit myself here to the so called *minimal model* [17].

It involves the following Higgs representations

$$H(\mathbf{10}), \quad \Phi(\mathbf{210}), \quad \Sigma(\mathbf{126}) \oplus \overline{\Sigma}(\overline{\mathbf{126}}) .$$

Both Σ and $\overline{\Sigma}$ are required to avoid high scale $SUSY$ breaking (D -flatness) and $\Phi(\mathbf{210})$ needed for the gauge breaking.

The properties of the model are dictated by the superpotential. This involves all possible renormalizable products of the superfields

$$W = M_\Phi \Phi^2 + \lambda_\Phi \Phi^3 + M_\Sigma \Sigma \overline{\Sigma} + \lambda_\Sigma \Phi \Sigma \overline{\Sigma} \\ + M_H H^2 + \Phi H (\kappa \Sigma + \bar{\kappa} \overline{\Sigma}) + \Psi_i (Y_{10}^{ij} H + Y_{\overline{126}}^{ij} \overline{\Sigma}) \Psi_j$$

(One can, however, add discrete symmetries or $U(1)_{PQ}$ etc. on top of $SO(10)$.)

We take all coupling constants real and positive, also in the soft $SUSY$ breaking terms.

The symmetry breaking goes in two steps

$$SUSY SO(10) \xrightarrow{\text{strong gauge breaking}} MSSM \xrightarrow{SUSY \text{ breaking}} SM$$

The F and D -terms must vanish during the strong gauge breaking to avoid high scale $SUSY$ breakdown ("F,D flatness").

D -flatness: only $\Sigma, \bar{\Sigma}$ are relevant therefore

$$|\Delta| = |\bar{\Delta}| \text{ i.e. } \sigma = \bar{\sigma} .$$

The situation with F -flatness is more complicated.

The strong breaking is dictated by the VEV s that are SM singlets.

Those are in the $SU_C(4) \times SU_L(2) \times SU_R(2)$ notation :

$$\begin{aligned} \phi_1 = < \Phi(1, 1, 1) > \quad \phi_2 = < \Phi(15, 1, 1) > \quad \phi_3 = < \Phi(15, 1, 3) > \\ \Delta = < \Sigma(\bar{10}, 1, 3) > \quad \bar{\Delta} = < \bar{\Sigma}(10, 1, 3) > . \end{aligned}$$

The strong breaking superpotential in terms of those VEV 's is then

$$\begin{aligned} W_H = & M_\phi(\phi_1^2 + 3\phi_2^2 + 6\phi_3^2) + 2\lambda_\phi(\phi_1^3 + 3\phi_1\phi_2^2 + 6\phi_2\phi_3^2) \\ & + M_W\Delta\bar{\Delta} + \lambda_\Sigma\Delta\bar{\Delta}(\phi_1 + 3\phi_2 + 6\phi_3). \end{aligned}$$

$\left| \frac{\partial W_H}{\partial v_i} \right|^2 = 0$ gives a set of equations. Their solutions dictate the details of the strong symmetry breaking. One chooses the parameters such that the breaking

$$SUSY SO(10) \longrightarrow MSSM$$

will be achieved [18] [19].

$SUSY$ is broken by the soft $SUSY$ breaking terms. The gauge $MSSM$ breaking is induced by the VEV 's of the SM doublet $\phi^{u,d}(1, 2, \pm 1/2)$ components of the Higgs representations.

The mass matrices of the Higgs are then as follows

$$M_{ij}^u = \left[\frac{\partial^2 W}{\partial \phi_i^u \partial \phi_j^u} \right]_{\phi_i = < \phi_i >} \quad M_{ij}^d = \left[\frac{\partial^2 W}{\partial \phi_i^d \partial \phi_j^d} \right]_{\phi_i = < \phi_i >} .$$

The requirement

$$\det(M_{ij}^u) \approx 0 \quad \det(M_{ij}^d) \approx 0$$

leaves only two light combinations of doublet components and those play the role of the bidoublets h_u, h_d of the $MSSM$. (This also is discussed in detail in the papers of [18] [19].)

We will come back to h_u, h_d later but let me discuss the $SCPV$ first.

Spontaneous CP violation in $SUSY$ $SO(10)$

As in the non-SUSY case, we conjecture that Δ and $\bar{\Delta}$, and only those, acquire a phase at the tree level

$$\langle \Sigma(1, 1, 0) \rangle \equiv \Delta = \sigma e^{i\alpha} \quad \langle \bar{\Sigma}(1, 1, 0) \rangle \equiv \bar{\Delta} = \sigma e^{i\bar{\alpha}}.$$

Let me show that this is a minimum of the scalar potential in a certain region of the parameter space.

To do this we collect all terms with $\Delta, \bar{\Delta}$ in the superpotential. Those involve the VEV 's that are non-singlets under the SM . I.e. the SM doublet components of the Higgs representations.

$$\begin{aligned} \phi^u &= \langle \Phi(1, 2, 1/2) \rangle & \phi^d &= \langle \Phi(1, 2, -1/2) \rangle \\ H^u &= \langle H(1, 2, 1/2) \rangle & H^d &= \langle H(1, 2, -1/2) \rangle \\ \Delta^u &= \langle \Sigma(1, 2, 1/2) \rangle & \Delta^d &= \langle \Sigma(1, 2, -1/2) \rangle \\ \bar{\Delta}^u &= \langle \bar{\Sigma}(1, 2, 1/2) \rangle & \bar{\Delta}^d &= \langle \bar{\Sigma}(1, 2, -1/2) \rangle \end{aligned}$$

The relevant terms are:

$$\begin{aligned} W_\Delta &= M_\Sigma \Delta \bar{\Delta} + \frac{\lambda_\Sigma}{10} (\phi^u \Delta^d \bar{\Delta} + \phi^d \bar{\Delta}^u \Delta) \\ &+ \left(\frac{\lambda_\Sigma}{10} \left(\frac{1}{\sqrt{6}} \phi_1 \Delta \bar{\Delta} + \frac{1}{\sqrt{2}} \phi_2 \Delta \bar{\Delta} + \phi_3 \Delta \bar{\Delta} \right) \right. \\ &+ \left. \frac{\lambda_\Sigma \sqrt{2}}{15} \phi_2 \bar{\Delta}^u \Delta^d - \frac{\kappa}{\sqrt{5}} \phi^d H^u \Delta - \frac{\bar{\kappa}}{\sqrt{5}} \phi^u H^d \bar{\Delta} \right) \end{aligned}$$

using [18] [19].

One can then calculate the corresponding scalar potential

$$V(\alpha, \bar{\alpha}, M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) = \sum_i \left| \frac{\partial W_\Sigma}{\partial v_i} \right|^2.$$

Noting that $|A + B e^{i\alpha}|^2 = A^2 + B^2 + 2AB \cos \alpha$

and $|K + P \Delta \bar{\Delta}|^2 = K^2 + P^2 \sigma \bar{\sigma} + 2KP \sigma \bar{\sigma} \cos(\alpha + \bar{\alpha})$,

one finds that

$$V = A(M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) + B(M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) \cos \alpha +$$

$$D(M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) \cos \bar{\alpha} + E(M_\Sigma, \lambda_\Sigma, \kappa, \bar{\kappa}, v_i) \cos(\alpha + \bar{\alpha}).$$

For explicit expressions of the coefficients see the Appendix.

The minimalization under $\alpha, \bar{\alpha}$ requires

$$\begin{aligned} \frac{\partial V(\alpha)}{\partial \alpha} &= -B \sin \alpha - E \sin(\alpha + \bar{\alpha}) = 0 \\ \frac{\partial V(\bar{\alpha})}{\partial \bar{\alpha}} &= -D \sin \bar{\alpha} - E \sin(\alpha + \bar{\alpha}) = 0 \end{aligned}.$$

This gives the equations

$$\sin \bar{\alpha} = \frac{B}{D} \sin \alpha$$

$$B \sin \alpha + E(\sin \alpha \cos \bar{\alpha} + \sin \bar{\alpha} \cos \alpha) = 0$$

and the solutions are

$$\cos \alpha = \frac{ED}{2} \left(\frac{1}{B^2} - \frac{1}{D^2} - \frac{1}{E^2} \right)$$

$$\cos \bar{\alpha} = \frac{EB}{2} \left(\frac{1}{D^2} - \frac{1}{B^2} - \frac{1}{E^2} \right).$$

We have clearly a minimum for a certain range of parameters, with non trivial values of $\alpha, \bar{\alpha}$. This means that CP is broken spontaneously.

Our $SCPV$ induces a phase in the CKM matrix

We mentioned already that the $MSSM$ bi-doublets h_u, h_d are given (linear) combinations of the Higgs representations doublet components. The general explicit combination are given in [18] and partially also in [19]. Those expressions are quite complicate so let me skip them and refer you to the above papers.

The important relevant fact for us is that coefficients of those combinations involve Δ and $\bar{\Delta}$ (and a possibly complex parameter x that fixes the local symmetry breaking [18]) so that the $VEVs$ $\langle h_u \rangle, \langle h_d \rangle$ are complex.

H and $\bar{\Sigma}$ which come in the Yukawa coupling and contribute to the mass matrices

$$M^i = Y_{10}^i H + Y_{\overline{126}}^i \bar{\Sigma}$$

are given in terms of the physical $h_{u,d}$ as follows (the heavy combinations decouple):

$$\begin{aligned} H_{u,d} &= a_u h_u + a_d h_d + \dots \text{ decoupled} \\ \bar{\Sigma}_{u,d} &= b_u h_u + b_d h_d + \dots \text{ decoupled} \end{aligned}$$

The mass matrices are expressed then in terms of $\langle h_{u,d} \rangle$

$$\begin{aligned} M_u &= (a_u Y_{10} + b_u Y_{\overline{126}}) \langle h_u \rangle \\ M_d &= (a_d Y_{10} + b_d Y_{\overline{126}}) \langle h_d \rangle \\ M_\ell &= (a_d Y_{10} - 3b_d Y_{\overline{126}}) \langle h_d \rangle \\ M_\nu^D &= (a_u Y_{10} - 3b_u Y_{\overline{126}}) \langle h_u \rangle \\ M_{\nu_R} &= Y_{\overline{126}} \bar{\Delta} \end{aligned}$$

The mass matrices of the quarks and also leptons are therefore complex and lead to a complex CKM matrix as well as a complex $PNMS$ leptonic one.

Remarks concerning other *SCPV* models

To the best of my knowledge there are no *SUSY GUT* models that really discuss the way the phases are generated spontaneously. *SCPV* is induced in most models in giving adhoc phases by hand to some of the *VEV*s.

Is the *SCPV* related to the strong *CP* problem?

The spontaneous breaking of *CP* solves the *QCD* Θ problem but only at the tree level. To suppress also radiative corrections, *a la* Barr [20] and Nelson [21], one must however go beyond *SO*(10). The simplest solution, in the framework of the renormalizable *SO*(10), is to require global *U*(1)_{*PQ*} [9] invariance with the invisible axion scenario [22]. It is interesting then to observe that the energy range of our *SCPV* lies within the invisible axion window [4]

$$10^9 GeV \lesssim f_a \lesssim 10^{12} GeV ,$$

where f_a is the axion decay constant.

This can be applied to *SUSY SO*(10) as well. The minimal renormalizable *SUSY SO*(10) \times *U*(1)_{*PQ*} was discussed recently in a paper by Fukuyama and Kikuchi [23]. The requirement of *U*(1)_{*PQ*} invariance using the *PQ* charges

$$PQ(\Psi) = -1, \quad PQ(H) = 2,$$

$$PQ(\Sigma) = -2, \quad PQ(\bar{\Sigma}) = 2, \quad PQ(\Phi) = 0$$

forbids only two terms in the superpotential

$$W_{PQ} = M\Phi^2 + \lambda_\Phi \Phi^3 + M_\Sigma \Sigma \bar{\Sigma} + \lambda_\Sigma \Phi \Sigma \bar{\Sigma} \\ + \mathcal{K} \Phi \Sigma H + \Psi_i (Y_{10}^{ij} H + Y_{126}^{ij} \bar{\Sigma}) \Psi_j \quad .$$

Hence, our scenario for *SCPV* is still intact (although with different phases).

The breaking of local *B* – *L* via the *VEV*s of $\bar{\Sigma}(\overline{126})$ and $\Sigma(126)$ will also break spontaneously the global *U*(1)_{*PQ*} and explain the coincidence of the scales of the axion window and the seesaw one. In our scenario it will also coincide with the scale of *SCPV* and that of leptogenesis.

Fukuyama and Kikuchi [23] suggest in their paper that the difference between the phases of Δ and $\bar{\Delta}$ is related to the axion¹.

¹G.Senjanovic claims however that it is not possible to break two symmetries using one *VEV* (private communication after my talk in Paris).

Conclusions

I presented, in these talks, a scenario for *SCPV* in both non-*SUSY* and *SUSY SO(10)*. *CP* is broken spontaneously at the scale of the *RH* neutrinos but a phase is generated also in the *CKM* low energy mixing matrix. We have therefore *CP* violation at low and high energies as is required experimentally.

If $U(1)_{PQ}$ invariance is also used, one finds the interesting situation that the scales of Baryogenesis, Seesaw, *SCPV* and the breaking of $U(1)_{PQ}$ are all at the same order of magnitude.

A detailed paper based on the above talks is in preparation.

Appendix: the parameters of the scalar potential

$$\frac{\partial W_\Delta}{\partial \phi^u}, \quad \frac{\partial W_\Delta}{\partial \phi^d}, \quad \frac{\partial W_\Delta}{\partial \phi_1}, \quad \frac{\partial W_\Delta}{\partial \phi_3}, \quad \frac{\partial W_\Delta}{\partial H^u}, \quad \frac{\partial W_\Delta}{\partial H^d}$$

do not give terms with a phase.

α dependent terms are obtained from $\frac{\partial W_\Delta}{\partial \Delta}$ and $\frac{\partial W_\Delta}{\partial \Delta^u}$ i.e.

$$\left| \frac{\partial W_\Delta}{\partial \bar{\Delta}} \right|^2 + \left| \frac{\partial W_\Delta}{\partial \bar{\Delta}^u} \right|^2 = \text{constant} + B \cos \alpha$$

Therefore,

$$B = 2\sigma\phi^u \left[M_\Sigma + \frac{\lambda_\Sigma}{10} \left(\frac{1}{\sqrt{6}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2 + \phi_3 \right) \right] \left[\frac{\lambda_\Sigma}{10}\Delta^d - \frac{\bar{\kappa}}{\sqrt{5}}H^d \right] + \frac{\sqrt{2}}{75}\sigma\lambda_\Sigma^2\phi^d\phi_2\Delta^d =$$

$$B(M_\Sigma, \lambda_\Sigma, \bar{\kappa}, \phi_i, \phi^u, \phi^d, \Delta^d, H^d) .$$

In the same way

$$D = 2\sigma\phi^d \left[M_\Sigma + \frac{\lambda_\Sigma}{10} \left(\frac{1}{\sqrt{6}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2 + \phi_3 \right) \right] \left[\frac{\lambda_\Sigma}{10}\Delta^u - \frac{\bar{\kappa}}{\sqrt{5}}H^u \right] + \frac{\sqrt{2}}{75}\sigma\lambda_\Sigma^2\phi^u\phi_2\Delta^u =$$

$$D(M_\Sigma, \lambda_\Sigma, \kappa, \phi_i, \phi^u, \phi^d, \Delta^u, H^u) .$$

A term proportional to $\cos(\alpha + \bar{\alpha})$ is generated only by $\frac{\partial W_\Delta}{\partial \phi_2}$. Hence,

$$E = \frac{1}{75}\lambda_\Sigma^2\bar{\Delta}^u\Delta^d\sigma^2 = E(\lambda_\Sigma, \sigma, \bar{\Delta}^u, \Delta^d) .$$

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